Qualifying Exam Syllabus

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March 25th, 10am (on Zoom)

Committee: Nikhil Srivastava (chair), Bernd Sturmfels (advisor), Vera Serganova, Serkan Hoşten.

1 Major topic: Representation Theory (Algebra)

References: *A Journey Through Representation Theory* by Gruson and Serganova (Chapters 1, 3, 7, Chapter 2 – Sections 6-8), *Representation Theory* by Fulton and Harris (Part I, Sections 1-4).

Representations of finite groups: representations, invariant subspaces, irreducible representations, isomorphism, ways to produce a new representation, Maschke's theorem, Schur's lemma, complete reducibility, induced representations, Frobenius reciprocity, Mackey's criterion, Wedderburn-Artin theorem.

Character theory: character, dimension, orthogonality relations, number of irreducible representations of a finite group, isotypic components.

Representations of compact groups: compact groups, Haar measure, orthogonality relations, matrix coefficients, Peter-Weyl theorem.

Representations of symmetric groups: Young [diagrams, tableaux, symmetrizer], Schur-Weyl duality, symmetric algebra, monomial basis, Kostka numbers, standard and semi-standard Young tableaux, Young lattice, Schur polynomial, Pieri's rule, Frobenius formula, hook formula.

2 Major topic: Nonlinear Algebra (Algebra)

References: *Invitation to Nonlinear Algebra* by Michałek and Sturmfels (Chapters 1-8, 11-13).

Polynomials: ideals, Gröbner bases, Hilbert [function, series, polynomial], dimension, degree.

Varieties: affine varieties, Zariski topology, Bézout's theorem, projective varieties.

Solving and decomposing: zero-dimensional ideals, primary decomposition, linear PDE with constant coefficients.

Mapping and projecting: elimination, Sylvester matrix, resultants, image of a polynomial map.

Linear spaces and Grassmannians: coordinates for linear spaces, Plücker relations, Grassmannian, Schubert calculus, degree of Grassmannian.

Nullstellensatz: certificates for infeasibility, Hilbert's Nullstellensatz, combinatorial Nullstellentsatz, real Nullstellensatz, Artin's Theorem.

Tropical Algebra: arithmetic and valuations, linear algebra over the tropical semiring, tropical varieties.

Toric Varieties: algebraic torus, character, affine toric varieties, projective toric varieties, varieties from polytopes.

Invariant Theory: Reynolds operator, Hilbert's finiteness theorem, classical and geometric invariant theory, Derksen's algorithm.

Semidefinite Programming: spectrahedra, semidefinite programming, duality, sums of squares.

Combinatorics: matroids, lattice polytopes, generating functions.

3 Minor topic: Graph Theory (Applied math)

References: *Introduction to Graph Theory* by West (Chapters 1-3), *Modern Graph Theory* by Bollobás (Chapters 2, 9).

Basic notions: graphs, adjacency matrix, isomorphism, connected graphs, bipartite graphs, Eulerian curcuits, trees, spanning trees, deletion-contraction formula, Kruskal's algorithm, distance.

Electrical networks: graphs and electrical networks, Kirchhoff's laws, squaring the square, vector spaces and matrices associated with graphs, Kirchhoff matrix.

Matching: maximum matching, matching in bipartite graphs, Hall's matching theorem, independent sets and covers, min-max theorems (König), augmented path algorithm.

Random walks on graphs: Thomson's principle, Dirichlet's principle, electrical networks and random walks, hitting times and commute times, conductance and rapid mixing.